LABORATORY VIII
MECHANICAL OSCILLATIONS

Most of the laboratory problems so far have involved objects moving with constant acceleration because the total force acting on those objects was constant. In this set of laboratory problems, the total force acting on an object, and thus its acceleration, will change with position. When the position and the acceleration of an object change in a periodic manner, we say that the object undergoes oscillations.

You are familiar with many objects that oscillate, such as pendula and the strings of a guitar. At the atomic level, atoms oscillate within molecules, and molecules oscillate within solids. This molecular oscillation gives an object the internal energy that defines its temperature. Springs are a common example of objects that exert the type of force that will cause oscillatory motion.

In this lab you will study oscillatory motion caused by springs exerting a changing force on an object. You will use different methods to determine the *strength* of the total force exerted by different spring configurations, and you will investigate what determines a system's oscillation *frequency*.

OBJECTIVES:

After successfully completing this laboratory, you should be able to:

- Provide a qualitative explanation of the behavior of oscillating systems using the concepts of restoring force and equilibrium position.
- Identify the physical quantities that influence the period (or frequency) of the oscillatory motion and describe this influence quantitatively.
- Describe qualitatively the effect of additional forces on an oscillator's motion.

PREPARATION:

Read Serway & Vuille Chapter 13 sections 13.1, 13.2, and 13.3

Before coming to lab you should be able to:

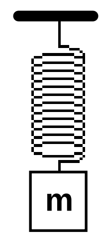
- Describe the similarities and differences in the behavior of the sine and cosine functions.
- Recognize the difference between amplitude, frequency, and period for repetitive motion.
- Determine the force on an object exerted by a spring using the concept of a spring constant.

PROBLEM #1:
MEASURING SPRING CONSTANTS

You are selecting springs for use in a large antique clock. In order to determine the force that they exert when stretched, you need to know their spring constants. One book recommends a **static approach**, in which objects of different weights hang from the spring and the displacement from equilibrium is measured. Another book suggests a **dynamic approach**, in which an object hanging from the end of a spring is set into motion and its oscillation frequency is measured. You wish to determine if these different approaches yield the same value for the spring constant. You decide to take both static and dynamic measurements and then compare.

EQUIPMENT

You will have a spring, a table clamp and metal rod, assorted masses, a mass hanger, a meter stick, a triple-beam balance, and a stopwatch.



PREDICTION

1. Write an expression for the relationship between the spring constant and the displacement of an object hanging from a spring.

2. Write an expression for the relationship between the spring constant and the period of oscillation of an object hanging from a spring.

WARM-UP

Read Serway & Vuille, Chapter 13, Sections 13.1 and 13.2

Method #1 (Static Approach)

1. Make two pictures of the situation, one before you attach an object to a spring, and one after an object is suspended from the spring and is at rest. Draw a coordinate system. On each picture, label the position where the spring is unstretched, the distance from the unstretched position to the stretched position, the mass of the object, and the spring constant.

2. Draw a force diagram for an object hanging from a spring at rest. Label the forces acting on the object. Use Newton's second law to write the equation of motion for the object.

3. Solve the equation of motion for the spring constant in terms of the other values in the equation. What does this tell you about the slope of a displacement (from the unstretched position) versus weight of the object graph?

Method #2 (Dynamic Approach): Suppose you hang an object from the spring, start it oscillating, and measure the *period* of oscillation.

1. Make three pictures of the oscillating system: (1) when the mass is at it maximum displacement *below* its equilibrium position, (2) after one half period, and (3) after one period. On each picture put arrows to represent the object's velocity and acceleration.
2. Write down an equation that is the relationship between the object's period, its mass, and the spring constant. Solve the equation for the spring constant in terms of the object's mass and period.

EXPLORATION

Method #1 - Static Approach:
Select a series of masses that give a usable range of displacements**.** The largest mass should not pull the spring past its elastic limit, for two reasons: (1) beyond the elastic limit there is no well-defined spring constant, and (2) a spring stretched beyond the elastic limit will be damaged.

Clamp the metal rod to the table, and hang the spring from the rod. Decide on a procedure that allows you to measure the distance a spring stretches when an object hangs from it in a consistent manner. Decide how many measurements you will need to make a reliable determination of the spring constant.

Method #2 - Dynamic Approach:
Secure the spring to the metal rod and select a mass that gives a regular oscillation without excessive wobbling. The largest mass you choose should not pull the spring past its elastic limit and the smallest mass should be much greater than the mass of the spring. Practice starting the mass in motion smoothly and consistently.

Decide how to measure the period of oscillation of the object-spring system most accurately. How can you minimize the uncertainty introduced by your reaction time in starting and stopping the stopwatch? How many times should you measure the period to get a reliable value? How will you determine the uncertainty in the period?

MEASUREMENT

For both methods, make the measurements that you need to determine the spring constant. **DO NOT STRETCH THE SPRINGS PAST THEIR ELASTIC LIMIT (ABOUT 60 CM) OR YOU WILL DAMAGE THEM.** Analyze your data as you go along so you can decide how many measurements you need to make to determine the spring constant accurately and reliably with each method.

ANALYSIS

Method #1: Graph displacement versus weight for the object-spring system. From the slope of this graph, calculate the value of the spring constant. Estimate the uncertainty in this measurement of the spring constant.

Method #2: Graph period versus mass for the object-spring system. If this graph is not a straight line, use *Appendix C: How do I linearize my data?* as a guide to linearize the graph. From the slope of the straight-line graph, calculate the value of the spring constant. Estimate the uncertainty in this measurement of the spring constant.

CONCLUSION

For each method, does the graph have the characteristics you predicted? How do the values of the spring constant compare between the two methods? Which method do you feel is the most reliable? Justify your answers.

PROBLEM #2:
EFFECTIVE SPRING CONSTANT

Your company has bought the prototype for a new flow regulator from a local inventor. Your job is to prepare the prototype for mass-production. While studying the prototype, you notice the inventor used some rather innovative spring configurations to supply the tension needed for the regulator valve. In one location the inventor had fastened two different springs side-by-side, as in Figure A below. In another location the inventor attached two different springs end-to-end, as in Figure B below.

To decrease the cost and increase the reliability of the flow regulator for mass production, you need to replace each spring configuration with a single spring. These replacement springs must exert the same forces when stretched the same amount as the original spring configurations.

EQUIPMENT

You have two different springs that have the same unstretched length, but different spring constants k_1 and k_2. These springs can be hung vertically side-by-side (Figure A) or end-to-end (Figure B).

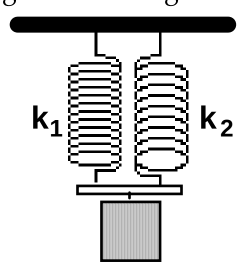


Figure A

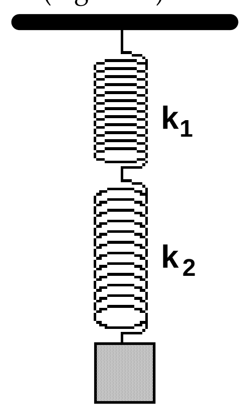


Figure B

As in Problem #1, you will have a table clamp and metal rod, a meter stick, a mass holder, assorted masses, a balance, and a stopwatch.

PREDICTIONS

The spring constant for a single spring that replaces a configuration of springs is called its *effective spring constant*.

1. Write an expression for the effective spring constant for a side-by-side spring configuration (Figure A) in terms of the two spring constants k_1 and k_2.
2. Write an expression for the effective spring constant for an end-to-end spring configuration (Figure B) in terms of the two spring constants k_1 and k_2.

Is the effective spring constant larger when the two springs are connected side-by-side or end-to-end? Explain your reasoning.

WARM-UP

Read Serway & Vuille, Chapter 13, Sections 13.1 and 13.2

Apply the following warm-up to the side-by-side configuration, and then repeat for the end-to-end configuration:

1. Make a picture of the spring configuration similar to each of the drawings in the Equipment section (Figure A and Figure B). Draw a coordinate system. Label the positions of each unstretched spring, the final stretched position of each spring, the two spring constants, and the mass of the object suspended. Put arrows on your picture to represent any forces on the object. Assume that the springs are massless.

For the side-by-side configuration, assume that the light bar attached to the springs remains horizontal (i.e. it does not twist).

For each two spring configurations make a second picture of a single (massless) spring with spring constant k' that has the same object suspended from it and the same total stretch as the combined springs. Be sure to label this picture in the same manner as the first.

2. Draw force diagrams of both spring systems and the equivalent single spring system. Label the forces. For the end-to-end configuration, draw an additional force diagram of a point at the connection of the two springs.

3. Apply Newton's laws to the object suspended from the combined springs and the object suspended from the single replacement spring. Consider carefully which forces and displacements will be equal to each other

For the end-to-end configuration: Draw an additional force diagram for the connection point between the springs. At the connection point, what is the force of the top spring on the bottom spring? What is the force of the bottom spring on the top spring?

4. Solve your equations for the effective spring constant (k') for the single replacement spring in terms of the two spring constants.

EXPLORATION

To test your predictions, you must decide how to measure each spring constant of the two springs and the effective spring constants of the side-by-side and end-to-end configurations.

From your results of Problem #1, select the best method for measuring spring constants (the static or dynamic method). Justify your choice.

Perform an exploration consistent with your selected method. If necessary, refer back to the appropriate Exploration section of Problem #1. Remember that the smallest mass must be much greater than the mass of the spring to fulfill the massless spring assumption. **DO NOT STRETCH THE SPRINGS PAST THEIR ELASTIC LIMIT (ABOUT 60 CM) OR YOU WILL DAMAGE THEM.**

Write down your measurement plan.

MEASUREMENT

Follow your measurement plan to take the necessary data. If necessary, refer back to the appropriate Measurement section of Problem #1. What are the uncertainties in your measurements?

ANALYSIS

Determine the effective spring constants (with uncertainties) of the side-by-side spring configuration and the end-to-end spring configuration. If necessary, refer back to Problem #1 for the analysis technique consistent with your selected method.

Determine the spring constants of the two springs. Calculate the effective spring constants (with uncertainties) of the two configurations using your Prediction equations.

CONCLUSION

How do the measured values and predicted values of the effective spring constant for the configurations compare?

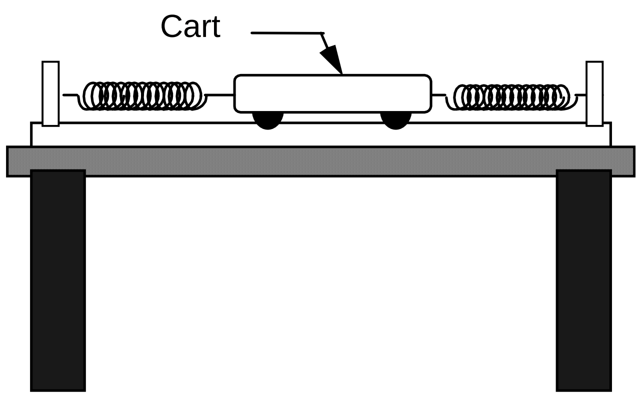
What are the effective spring constants of a side-by-side spring configuration and an end-to-end spring configuration? Which is larger? Did your measured values agree with your initial predictions? Why or why not? What are the limitations on the accuracy of your measurements and analysis? Can you apply what you learned to find the spring constant of a complex system of springs in the flow regulator?

PROBLEM #3:
OSCILLATION FREQUENCY WITH TWO SPRINGS

You have a summer job with a research group at the University. Because of your physics background, your supervisor asks you to design equipment to measure earthquake aftershocks. A calibration sensor needs to be isolated from the earth movements, yet it must be free to move. You decide to place the sensor on a low friction cart on a track and attach a spring to both sides of the cart. To make any quantitative measurements with the sensor you need to know the frequency of oscillation for the cart as a function of the spring constants and the mass of the cart.

EQUIPMENT

You will have an aluminum track, a PASCO cart, two adjustable end stops, two springs, a meter stick, and a stopwatch. You will also have a video camera and a computer with video analysis applications written in LabVIEW™ (VideoRECORDER and VideoTOOL).



PREDICTION

Write an expression for the frequency of the cart in terms of its mass and the two spring constants.

WARM-UP

Read Serway & Vuille, Chapter 13, Sections 13.1 and 13.2

1. Make two pictures of the oscillating cart (1) one at its equilibrium position and (2) one at some other position and time while it is oscillating. On your pictures, show the direction of the velocity and acceleration of the cart and the forces on the cart.

2. Draw a force diagram of the oscillating cart when it is at a position away from its equilibrium position. Label the forces.

3. Write down an equation for the total force on the cart in terms of the two spring constants and its displacement from the equilibrium position.

4. Now imagine that only one spring was attached to the cart, but it exerted the same force at the same displacement as the two-spring system. How would the motion of these two systems compare? What is the relationship between the spring constant of the single spring system and the two for the two-spring system?

5. Write down an equation for the frequency of the imaginary one spring system. How does it compare with the frequency of the two-spring system?

6. Use the simulation "LabSim5" (See *Appendix F* for a brief explanation of how to use the simulations) to approximate the conditions of this problem.

EXPLORATION

Decide the best method to determine the spring constants based on your results of Problem #1. **DO NOT STRETCH THE SPRINGS PAST THEIR ELASTIC LIMIT (ABOUT 60 CM) OR YOU WILL DAMAGE THEM.**

Find the best place for the adjustable end stop on the track. *Do not stretch the springs past 60 cm*, but stretch them enough so the cart oscillates smoothly. Find the most appropriate cart mass.

Practice releasing the cart smoothly. How long does it take for the oscillations to stop? What effect will this have on your measured values compared to your predicted values? How can you affect this time? What amplitude will you use to take your measurements? Between what positions should you measure a cycle? Over how many cycles should you measure to get a precise result?

MEASUREMENT

Determine the spring constants. Record these values. What is the uncertainty in these measurements?

Measure the period of oscillation of the cart. How many times should you take this measurement to be sure that it is reliable? What is the uncertainty in your measurement?

ANALYSIS

Analyze your video to find the period of oscillation. Calculate the frequency (with uncertainty) of the oscillations from your measured period. Calculate the frequency (with uncertainty) using your Prediction equation. How does your measured frequency compare with your predicted frequency?

CONCLUSION

What is the frequency of the oscillating cart? Did your measured frequency agree with your predicted frequency? Why or why not? What are the limitations on the accuracy of your measurements and analysis?

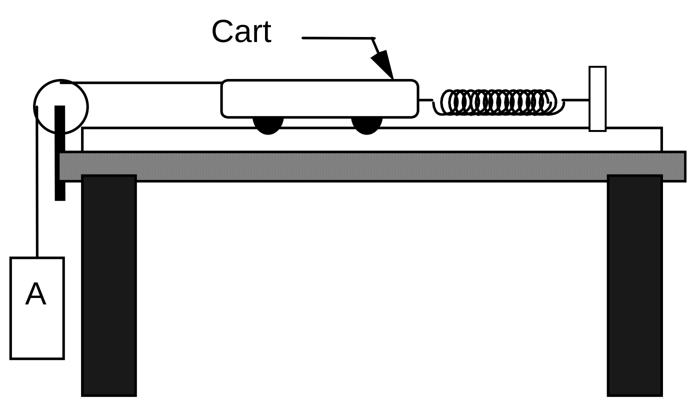
<u>If you completed Problem #2:</u> What is the effective spring constant of this configuration? How does it compare with the effective spring constants of the side-by-side and end-to-end configurations?

PROBLEM #4:
OSCILLATION FREQUENCY OF AN EXTENDED SYSTEM

You are the technical advisor for the next Bruce Willis action adventure movie, *Die Even Harder*, which is being filmed in Minnesota. The script calls for a spectacular stunt: Bruce Willis is dangling over a cliff from a long rope that is tied to the evil villain, who is on the ice-covered ledge of the cliff. The villain's *elastic* parachute line is tangled in a tree located several feet from the edge of the cliff. Bruce and the villain are in simple harmonic motion. At the top of his motion, Bruce unsuccessfully tries to grab for the cliff edge while the villain reaches for his boot knife. The script calls for the villain to cut the rope just as Bruce reaches the top of his motion again. It is expensive (and dangerous) to have Willis hanging from the rope while the crew films close-ups of the villain. However, the stunt-double weighs more than Bruce Willis. The director needs to know if the stunt double will have a different oscillatory motion than Bruce. You decide to solve this problem by modeling the situation with a cart on a track with a spring attached to one end and a hanging object attached to the other by a string. The track represents the ice-covered ledge of the cliff, the adjustable end-stop represents the tree, the spring represents the elastic parachute, the cart represents the villain, the string represents the rope, and the hanging object represents Bruce Willis or his stunt double in this problem.

EQUIPMENT

You have an aluminum track with an adjustable end-stop, a pulley that attaches to the end of the track, a spring, a PASCO cart, some string, a mass hanger, assorted masses, a meter stick, and a stopwatch.



PREDICTION

Write an expression for how the frequency of oscillation of the system depends on the mass of the object hanging over the table.

Use your equation to sketch the expected shape of a graph of oscillation frequency versus hanging mass. Will the frequency **increase**, **decrease** or **stay the same** as the hanging mass increases?

WARM-UP

Read Serway & Vuille, Chapter 13, Sections 13.1, 13.2 and 13.3

1. Make a picture of the situation when the cart and hanging object are at their equilibrium positions. Make another picture at some other time, while the system is oscillating. On your pictures, for the cart and the hanging object, show the directions of the acceleration and velocity. How are they related? Identify and label the known forces on the cart and on the hanging object.

2. Write down how to determine the frequency of a spring system's motion, if you know the relationship between the force on the object and the object's mass.

3. Draw separate force diagrams of the oscillating cart and hanging object when the system is not in its equilibrium position. Label the forces.

4. Independently apply Newton's laws to the cart and to the hanging object. Is the force of the string on the hanging mass constant?

5. Write down the equation that relates the acceleration of the cart to the forces on it. To determine the force of the string on the cart, use the same type of relationship for the hanging mass.

6. From the resulting equation that gives the acceleration of the cart in terms of the spring constant and masses, write down the frequency of oscillation of the cart. To determine the frequency, you should ignore any constant terms in this equation. Give an argument for doing this.

7. Use your equation to sketch the expected shape of the graph of oscillation frequency versus the hanging object's mass.

EXPLORATION

If you do not know the spring constant of your spring, you should decide the best way to determine the spring constant based on your results of Problem #1.

Find the best place for the adjustable end stop on the track. **DO NOT STRETCH THE SPRING PAST ITS ELASTIC LIMIT (ABOUT 60 CM) OR YOU WILL DAMAGE IT,** but stretch it enough so the cart and hanging mass oscillate smoothly.

Determine cart mass you will use for your measurements. Determine the best mass range for the hanging object.

Practice how you will release the cart-hanging mass system. How long does it take for the oscillations to stop? What effect will this have on your measured values compared to your predicted value? How can you affect this time? What amplitude will you use to take your measurements? Between what positions should you measure a cycle? Over how many cycles should you measure to get a precise result?

Write down your measurement plan.

MEASUREMENT

If necessary, determine the spring constant of your spring. What is the uncertainty in your measurement?

For each different hanging mass, measure the period of oscillation of the cart. How many times should you measure each period to be sure that it is reliable? What is the uncertainty of each measurement?

Analyze your data as you go along, so you can determine the size and number of different hanging masses you need to use.

Collect enough data to convince yourself and others of your conclusion about how the oscillation frequency depends on the hanging mass.

ANALYSIS

For each hanging mass, calculate the oscillation frequency (with uncertainty) from your measured period. Use your expression from the Prediction and Warm-up to calculate the *predicted* frequency for each mass.

Graph the frequency versus the hanging object's mass. On the same graph, show your predicted relationship.

What are the limitations on the accuracy of your measurements and analysis? Over what range of values does the measured graph match the predicted graph best? Do the two curves start to diverge from one another? If so, where? What does this tell you about the system?

CONCLUSION

Does the oscillation frequency increase, decrease, or stay the same as the hanging object's mass increases?

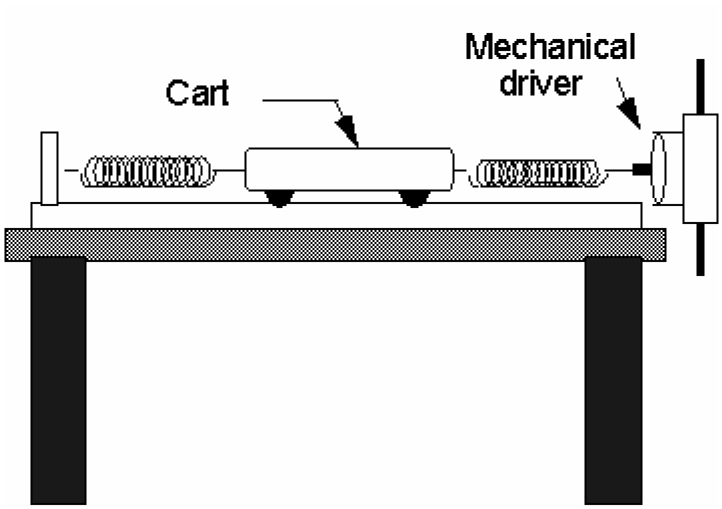
What will you tell the director? Do you think the motion of the actors in the stunt will change if the heavier stunt man is used instead of Bruce Willis? How much heavier than Bruce would the stunt man have to be to produce a noticeable difference in the oscillation frequency of the actors? Explain your reasoning in terms the director would understand.

EXPLORATORY PROBLEM #5:
DRIVEN OSCILLATIONS

You are now prepared to calibrate your seismic detector (from Problem #3). You need to determine how the amplitude of the oscillations of the detector will vary with the frequency of the earthquake aftershocks, so you replace the end stop on the track with a device that moves the end of the spring back and forth, simulating the earth moving beneath the track. The device, called a mechanical driver, is designed so you can change its frequency of oscillation.

EQUIPMENT

You will use a PASCO cart, two springs, an aluminum track, an adjustable end-stop, a signal generator, and a mechanical driver.



The mechanical driver is somewhat like a loudspeaker with one end of a metal rod attached to the center, and the other end of the rod is attached to one of the springs. The driver is connected to the signal generator that causes the rod to oscillate back and forth with adjustable frequencies that can be read off the display on the signal generator.

PREDICTION

Make your best-guess sketch of how you think a graph of the amplitude of the cart versus the frequency of the mechanical driver will look. Assume the driver has a constant amplitude of a few millimeters.

WARM-UP

Read Serway & Vuille, Chapter 13, Sections 13.1 and 13.2

Use the simulation "LabSim5" (See *Appendix F* for a brief explanation of how to use the simulations) to approximate the conditions of this problem.

EXPLORATION

Examine the mechanical driver. Mount it at the end of the track, using the clamp and metal rod so its shaft is aligned with the cart's motion. Connect it to the signal generator, using the output marked **Lo** (for ``low impedance''). Use the smallest amplitude that is sufficient to observe the oscillation of the cart at the lowest frequency possible.

Determine the accuracy of the digital display on the frequency generator by timing one of the lower frequencies.

Devise a scheme to accurately determine the amplitude of a cart on the track, and practice the technique. For each new frequency, should you restart the cart at rest?

When the driver is at or near the undriven frequency (natural frequency) of the cart-spring system, try to simultaneously observe the motion of the cart and the shaft of the driver. What is the relationship? What happens when the driver frequency is twice as large as that frequency?

MEASUREMENT

If you do not know the frequency of your system when it is not driven, determine it using the technique of Problem #3.

Collect enough cart amplitude and driver frequency data to test your prediction. Be sure to collect several data points near the undriven frequency of the system.

ANALYSIS

Make a graph the oscillation amplitude of the cart versus driver frequency.

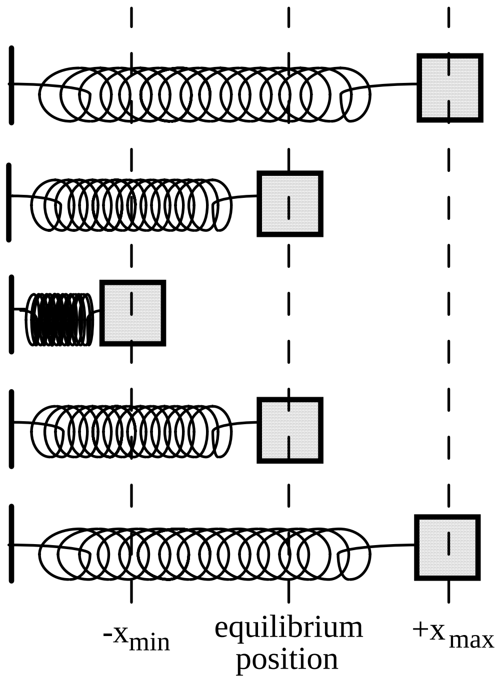
CONCLUSION

Is the graph what you had anticipated? Where is it different? Why? What are the limitations on the accuracy of your measurements and analysis?

Can you explain your results? Is energy conserved? What will you tell your boss about your design for a seismic detector?

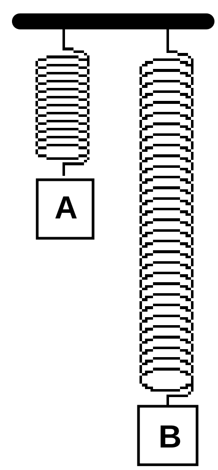
☑ CHECK YOUR UNDERSTANDING

1. The diagram below shows an oscillating mass/spring system at times 0, T/4, T/2, 3T/4, and T, where T is the period of oscillation. For each of these times, write an expression for the displacement (x), the velocity (v), the acceleration (a), the kinetic energy (KE), and the potential energy (PE) *in terms of the amplitude of the oscillations (A), the angular velocity (w), and the spring constant (k)*.



t	x	v	a	KE	PE
0					
$\frac{T}{4}$					
$\frac{T}{2}$					
$\frac{3T}{4}$					
T					

2. Identical masses are attached to identical springs that hang vertically. The masses are pulled down and released, but mass B is pulled further down than mass A, as shown at right.



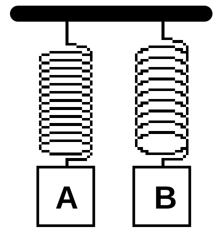
   a. Which mass will take a longer time to reach the equilibrium position? Explain.

   b. Which mass will have the greater acceleration at the instant of release, or will they have the same acceleration? Explain.

   c. Which mass will be going faster as it passes through equilibrium, or will they have the same speed? Explain.

   d. Which mass will have the greater acceleration at the equilibrium point, or will they have the same acceleration? Explain.

3. Two different masses are attached to different springs that hang vertically. Mass A is larger, but the period of simple harmonic motion is the same for both systems. They are pulled the same distance below their equilibrium positions and released from rest.



   a. Which spring has the greater spring constant? Explain.

   b. Which spring is stretched more at its equilibrium position? Explain.

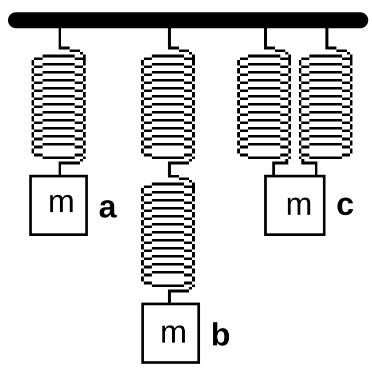
   c. The instant after release, which mass has the greater acceleration? Explain.

   d. If potential energy is defined to be zero at the equilibrium position for each mass, which system has the greater total energy of motion? Explain.

   e. Which mass will have the greater kinetic energy as it passes through its equilibrium position? Explain

   f. Which mass will have the greater speed as it passes through equilibrium? Explain.

4. Five identical springs and three identical masses are arranged as shown at right.



   a. Compare the stretches of the springs at equilibrium in the three cases. Explain.
   b. Which case, a, b, or c, has the greatest effective spring constant? The smallest effective spring constant? Explain.
   c. Which case would execute simple harmonic motion with the greatest period? With the least period? Explain.

   .

TA Name:______________________________

PHYSICS 1101 LABORATORY REPORT

Laboratory VIII

Name and ID#: ____

Date performed: ______________ Day/Time section meets: ________________________________

Lab Partners' Names: ____

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Problem # and Title: ____

Lab Instructor's Initials: _________

Grading Checklist	Points
LABORATORY JOURNAL:	
PREDICTIONS (individual predictions and warm-up completed in journal before each lab session)	
LAB PROCEDURE (measurement plan recorded in journal, tables and graphs made in journal as data is collected, observations written in journal)	
PROBLEM REPORT:*	
ORGANIZATION (clear and readable; logical progression from problem statement through conclusions; pictures provided where necessary; correct grammar and spelling; section headings provided; physics stated correctly)	
DATA AND DATA TABLES (clear and readable; units and assigned uncertainties clearly stated)	
RESULTS (results clearly indicated; correct, logical, and well-organized calculations with uncertainties indicated; scales, labels and uncertainties on graphs; physics stated correctly)	
CONCLUSIONS (comparison to prediction & theory discussed with physics stated correctly ; possible sources of uncertainties identified; attention called to experimental problems)	
TOTAL(incorrect or missing statement of physics will result in a maximum of 60% of the total points achieved; incorrect grammar or spelling will result in a maximum of 70% of the total points achieved)	
BONUS POINTS FOR TEAMWORK (as specified by course policy)	

* An "R" in the points column means to rewrite that section only and return it to your lab instructor within two days of the return of the report to you.